

# A global picture of quantum de Sitter space

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## Abstract

Perturbative gravity about a de Sitter background motivates a global picture of quantum dynamics in ‘eternal de Sitter space,’ the theory of states which are asymptotically de Sitter to both future and past. Eternal de Sitter physics is described by a finite dimensional Hilbert space in which *each* state is precisely invariant under the full de Sitter group. This resolves a previously-noted tension between de Sitter symmetry and finite entropy. Observables, implications for Boltzmann brains, and Poincaré recurrences are briefly discussed.

## I. INTRODUCTION

Modern cosmology provides ample motivation to understand de Sitter-like cosmologies in detail. The success of early universe inflation and current observations suggest that our universe emerged from an early high-scale de Sitter-like stage and is now at the onset of a low-scale stage. A feature of particular interest is the finite Bekenstein-Hawking entropy  $S_{dS}$  of the de Sitter (dS) horizon, which raises many questions about de Sitter microstates. It is often hoped that an understanding of this deep issue will have practical implications for the thorny problem of extracting predictions from a theory with eternal inflation, e.g. in the context of the string theory landscape.

Our work probes in such directions by exploring the perhaps simpler physics of ‘eternal de Sitter space,’ by which we mean the theory of states which are asymptotically de Sitter to both future and past with the *same* cosmological constant  $\Lambda$ . Our approach will be to investigate *perturbative* gravity about a de Sitter background in the regime where such a theory is self-consistent, and to use the results to motivate a deeper picture of eternal de Sitter quantum gravity.

Eternal de Sitter quantum gravity has been previously investigated by many authors. While there is no complete consensus, a common feature of many analyses has been to emphasize the region, centered on some freely falling worldline, in which de Sitter space may be described as a static spacetime. These analyses also emphasize the ‘static Hamiltonian’ which generates time translations in this ‘static patch.’ This emphasis has led various authors to suggest that dS physics is a close analogue to a static finite box at roughly the dS temperature  $T_{dS}$ , but with a finite number of states  $[1, 2, 3] \exp(S_{dS})$ .

However, it is difficult to find a rationale for such a static hamiltonian to be physical; certainly in a perturbative approach it appears unphysical. The reason for this is as follows. At zeroth order, the theory is just matter plus free gravitons on a de Sitter background. In this context, consider some ‘boost’ symmetry  $B$  of dS, which generates a corresponding symmetry of the zeroth order theory. In  $D$  spacetime dimensions,  $B$  fixes an  $S^{D-2}$  on the equator of  $S^{D-1}$ , and the Killing horizons of  $B$  are the cosmological horizons of two observers which we take to lie at the north and south poles of  $S^{D-1}$  as shown in figure 1. The boost  $B$  generates a symmetry of the order-zero theory, and this generator may be written as  $B = H_S - H_N$ , where  $H_N, H_S$  are the so-called static Hamiltonians of the associated north and south static patches.

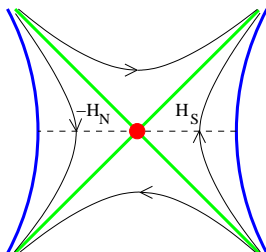


FIG. 1: De Sitter space with boost Killing field  $B = H_S - H_N$  and horizons. The dashed line is the ‘neck,’ the smallest  $S^{D-1}$  in this frame. The dot is the  $S^{D-2}$  left fixed by  $B$ . The worldline of the south (north) pole observer is the right (left) edge of the diagram.

The picture then changes at first non-trivial order in gravitational interactions. Here, so called linearization stability constraints [4, 5, 6, 7, 8, 9, 10] arise, which state that only zeroth

order states satisfying  $B = 0$  lead to consistent first order perturbations. The statement is akin to the fact that the total electric charge on a compact space must vanish. The point is that, at this (first) order, the *zeroth*-order boost generator  $B$  coincides with one of the gravitational Hamiltonian constraints and must vanish identically. While the action of  $B$  translates the static patch in time, we see that  $B$  generates a gauge symmetry and not a physically interesting notion of time evolution. In a perturbative quantum treatment, one requires that  $B$  annihilate physical states, so that the two static Hamiltonians  $H_S$  and  $H_N$  are perfectly correlated.

What do these facts imply for the action of  $H_S$ ? We may consider two contexts. First, as has become common, let us confine attention to the south patch and trace over degrees of freedom in the north. But due to the above correlations with  $H_N$ , the resulting density matrix  $\rho_S$  is diagonal in a basis of  $H_S$ -eigenstates. Thus,  $[\rho_S, H_S] = 0$  and  $H_S$  does not generate an useful notion of time evolution.

One might also consider the action of  $H_S$  on a region which overlaps the north patch. In doing so, we note that  $H_S$  alone does not generate a de Sitter symmetry. Instead, it generates the analogue of boosting the right Rindler wedge of Minkowski space while keeping the rest of Minkowski space fixed. Indeed, acting with  $H_S$  alone changes the short distance structure of the vacuum near the equatorial  $S^{D-2}$ . As a result, any action of  $H_S$  on such a region falls outside the scope of low energy physics. We conclude that the static Hamiltonian does not provide a useful basis for an analysis of de Sitter physics<sup>1</sup>.

In order to capture the effects noted above, we continue to address de Sitter physics below using perturbative gravity about the full de Sitter background. Our main focus (section II) is on the perturbative state space, but we also briefly address perturbative observables in section III. We take a global perspective, but confine ourselves to contexts where gravitational back-reaction is small. The above-mentioned linearization stability constraints play a key role in our analysis. Section IV then describes a picture of eternal de Sitter in full quantum gravity which is motivated by the results of sections II and III. Some remaining issues are discussed in section V. We plan to supply further details in future work.

## II. ETERNAL DE SITTER STATES

As noted in the introduction, the fact that Cauchy slices of de Sitter space are compact (spheres) leads to an interesting constraint on physical states, even when gravity is treated perturbatively. We explore the implications of such constraints below, and argue that eternal de Sitter physics is described by a finite-dimensional space in which each state is precisely de Sitter invariant.

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<sup>1</sup> An alternate interpretation is that a physical static Hamiltonian does exist, but that it is a genuinely new ingredient, visible only in the full quantum theory. This appears to be the viewpoint of [11], whose notion of a “static Hamiltonian” has eigenvalues bounded by the de Sitter temperature  $T_{dS}$ . This operator vanishes in the classical limit, and at this level is consistent with the arguments above. However, there remains some tension with the fact that such an operator does not appear at the semiclassical level, where  $T_{dS} \neq 0$ .

## A. de Sitter invariant states

Our context here is perturbative gravity. To be specific, consider the expansion of a gravitating theory in inverse powers of the Planck mass  $m_p$ , in which we take the graviton to have canonical normalization. At order zero, we have free gravitons together with a matter quantum field theory on a fixed de Sitter background. The matter theory may contain non-gravitational interactions of finite strength. For simplicity, we restrict attention to matter fields having a dS-invariant vacuum.

This order-zero theory has a large state space  $\mathcal{H}_0$ , but only certain states have consistent perturbative coupling to the gravitational field. At first order in  $1/m_p$ , one finds ‘linearization stability constraints’ [4, 5, 6, 7, 8, 9, 10] forcing each de Sitter charge  $Q[\xi] = \int_{\Sigma} T_{ab} n^a \xi^b$  to vanish. Here  $\xi$  is a de Sitter Killing field,  $\Sigma$  is a Cauchy surface, and  $Q[\xi]$  generates the corresponding isometry. Thus, only dS-invariant quantum states couple consistently to perturbative gravity; we refer to such states as ‘physical’ states. Similarly, only dS-invariant operators are observables. This is a direct analogue of a familiar feature of Maxwell gauge fields, where the presence of a compact Cauchy surface requires the total electric charge of any matter state to vanish. Thus, in the Maxwell context, allowed matter states are U(1) invariant, and only U(1) invariant operators will preserve this space of states.

While the order-zero Hilbert space ( $\mathcal{H}_0$ ) may have many states with zero electric charge, typically only the vacuum state will be fully de Sitter invariant [12]. As suggested in [12], one may nevertheless proceed by considering formally dS-invariant sums of states in  $\mathcal{H}_0$ , so long as one introduces a ‘renormalized’ inner product on the space of such sums. In particular, for each state  $|\psi\rangle$  in  $\mathcal{H}_0$ , consider the expression

$$|\Psi\rangle := \eta|\psi\rangle := \int dg U(g) |\psi\rangle, \quad (2.1)$$

where integral is over the de Sitter group and  $dg$  is the standard Haar measure. The result clearly satisfies the physical state condition<sup>2</sup>  $Q[\xi]|\Psi\rangle = 0$ . We will use the symbol  $\eta$  below to denote this ‘group averaging’ operation.

For physical states of the form (2.1), consider the new inner product:

$$\langle\Psi_1|\Psi_2\rangle := \langle\psi_1|\int dg U(g) |\psi_2\rangle. \quad (2.2)$$

For certain exactly solveable free theories, [12, 14] showed that (2.2) is finite when  $|\psi_1\rangle, |\psi_2\rangle$  are Fock basis states containing a sufficient number of particles<sup>3</sup> (typically 2 or 4). When completed with respect to the inner product (2.2), the space of group-averaged states forms

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<sup>2</sup> The formal expression (2.1) for  $|\Psi\rangle$  may be given meaning as a ‘distributional’ state analogous to the delta-function position or momentum eigenstates of quantum mechanics. In particular, for appropriate  $|\psi\rangle$ , expression (2.1) yields a well-defined linear function on an appropriate subspace of  $\mathcal{H}_0$ . We consider only states  $|\psi\rangle$  for which (2.1) converges in this sense. The dual action of  $Q[\xi]$  on  $|\Psi\rangle$  then annihilates  $|\Psi\rangle$ . See e.g. [13].

<sup>3</sup> The norm (2.2) is clearly *not* finite for a dS-invariant vacuum state  $|0\rangle$ , which must be treated separately. Such a separate treatment is justified by the fact that, for a well-defined observable  $\mathcal{O}$ , the state  $\mathcal{O}|0\rangle$  is dS-invariant and normalizable in  $\mathcal{H}_0$ ; i.e., it is proportional to  $|0\rangle$ . Thus  $\langle\psi|\mathcal{O}|0\rangle$  vanishes whenever  $|0\rangle$  and  $|\psi\rangle$  are orthogonal. (To avoid later confusion, we note that the action of (3.1) on  $|0\rangle$  does not yield a

the ‘physical’ Hilbert space  $\mathcal{H}_{phys}$ . This construction was also introduced independently in the general theory of constrained systems [18] and in the context of quantum cosmology [19]. The reader may consult the by now extensive literature (e.g. [13, 17, 20, 21, 22]) for more details. One may also motivate this approach via the functional integral. (See e.g. [23, 24] for related discussions.)

For (2.2) to be a valid inner product, it must be positive semi-definite. Ref. [12] showed that this is the case for a 1+1 free scalar toy model and for 3+1 linearized gravitons. While examples (in other contexts) are known for which group averaging is not positive definite [16], these cases are rather singular even at the classical level. Furthermore, when group averaging converges, one can show [21] that it gives the unique physical inner product consistent with the  $*$ -algebra of observables on  $\mathcal{H}_0$ ; i.e., if it fails to be positive definite, then *no* renormalized inner product will be positive. Below, we shall simply assume that (2.2) is positive when it converges, though an argument in appendix A below shows that (2.2) is indeed positive on a ‘large’ class of states.

## B. States with weak back-reaction

Perturbation theory can be valid only when gravitational interactions are weak. We should therefore confine attention to a appropriate subspace  $\mathcal{H}^w \subset \mathcal{H}_0$  of states which meet this criterion. Note that while such an  $\mathcal{H}^w$  will be de Sitter invariant as a linear space, the individual states in  $\mathcal{H}^w$  will not be dS-invariant. Group averaging will be required to transform them into physical states.

A variety of gravitational effects must be controlled for a state to lie in  $\mathcal{H}^w$ . For example, two energetic particles passing close to each other can result in a large gravitational scattering angle or in the formation of a black hole. Black hole formation may also result from the collective interaction of a large number of particles localized on a scale small compared to the de Sitter length scale  $R$ . However, these sorts of large gravitational effects also happen in the asymptotically flat context. On the other hand, a new gravitational effect for dS is that matter spread over an entire Cauchy surface can cause the overall acceleration of the universe to slow, and can even cause the spacetime to undergo a ‘big crunch.’ It is on this latter sort of effect that we wish to focus.

For the moment, let us consider a state  $|\psi\rangle \in \mathcal{H}_0$  of the matter field which is rotationally invariant under some  $SO(D)$  subgroup of the  $SO(D, 1)$  de Sitter group. This rotational symmetry picks out a preferred foliation of dS by spheres of symmetry. At early times the spheres are large and contract rapidly. The contraction gradually slows so that the spheres reach a minimum ‘neck’ of radius  $r = R$  and then begin to reexpand. For such states, the greatest danger of large gravitational back-reaction generically occurs at this neck, and the size of the gravitational back-reaction is controlled by the total flux of energy passing through the neck.

Let us therefore fix a foliation  $\mathcal{F}$  of dS by round spheres and consider the energy flux

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normalizeable state in  $\mathcal{H}_0$ ; the action of (3.1) is well-defined only on the orthogonal complement of  $|0\rangle$ .) See [13, 15, 16, 17] for comments on this issue. We are concerned here with generic states, and so will not treat the vacuum in detail.

operator

$$F = \int_{neck} \sqrt{{}^{D-1}g} T_{ab} n^a n^b, \quad (2.3)$$

where the integral is over the neck determined by the given foliation,  ${}^{D-1}g$  is the determinant of the induced metric,  $n^a$  is the unit future-pointing normal to the neck, and  $T_{ab}$  is a renormalized stress tensor such that  $\langle T_{ab} \rangle = 0$  in the dS-invariant vacuum.  $F$  is a well-defined operator with a discrete spectrum, free of accumulation points<sup>4</sup>. Let us also choose a cut-off  $f$ , such that a uniformly distributed energy density with  $F < f$  has small gravitational back-reaction. Dimensional analysis provides a rough estimate of the value of such  $f$  as the appropriate power of the de Sitter radius,

$$f \sim R^{D-3}/G, \quad (2.4)$$

where  $G \sim 1/m_p^{D-2}$ . Since the spectrum of  $F$  has no accumulation points, for a given foliation  $\mathcal{F}$  the bound  $F < f$  will be satisfied only by a finite number of quantum states. Because generic states with  $F < f$  are nearly uniform, and because we are most interested in the collective gravitational effects on the spacetime as a whole, we will neglect the fact that non-uniform states with  $F < f$  may still have large back-reaction. The states  $\mathcal{H}^{\mathcal{F},f}$  having  $F < f$  with respect to the given foliation  $\mathcal{F}$  (modulo those that are locally strongly coupled), comprise a subspace of  $\mathcal{H}^w$ .

On the other hand, there are many states in  $\mathcal{H}^w$  having large  $F$ , greater than  $f$ . In particular, the definition of  $F$  required a choice of foliation by spheres. Since the possible foliations are related by de Sitter transformations, which in general include de Sitter boosts, a state which has small  $F$  with respect to one foliation may have large  $F$  with respect to another. Since the de Sitter group is a gauge group in perturbative gravity, such states must also be considered to have small back-reaction. Physically, one notes that in a flat background the Aichelburg-Sexl metric is believed to give a largely accurate description of a highly-boosted particle, and one expects the same of the de Sitter version of this metric [25, 26].

We therefore take a state to lie in  $\mathcal{H}^w$  if it has  $F < f$  with respect to some foliation  $\mathcal{F}$ . In fact,  $\mathcal{H}^w$  will contain the space  $\mathcal{H}^f = \bigoplus_{\mathcal{F}} \mathcal{H}^{\mathcal{F},f}$  generated through arbitrary (normalizeable) superpositions of states in the individual  $\mathcal{H}^{\mathcal{F},f}$  for various foliations  $\mathcal{F}$ . Since one expects  $\mathcal{H}^f$  to include all states with weak back-reaction, we will use  $\mathcal{H}^f$  as a proxy for  $\mathcal{H}^w$  below.

Due to the freedom to choose any foliation  $\mathcal{F}$ , the space  $\mathcal{H}^f$  has infinite dimension. But the foliations differ only by dS transformations  $g$ , and group averaging takes states  $|\psi_1\rangle$  and  $U(g)|\psi_1\rangle$  to the same physical state. The dimension of the corresponding *physical* space  $\mathcal{H}_{phys}^f = \eta(\mathcal{H}^f)$  is thus no larger than that of a single  $\mathcal{H}^{\mathcal{F},f}$  for a fixed foliation. As stated above, this dimension is finite. In fact, using the techniques of [27] and assuming a typical relativistic equation of state  $p = \frac{\rho}{D-1}$ , the logarithm of the dimension of  $\mathcal{H}^{\mathcal{F},f}$  may be estimated roughly as  $(R/\ell_p)^{(D-1)(D-2)/D}$ , where  $\ell_p$  is the Planck length. In appendix A, it is shown that group averaging does not significantly change this result; i.e., the logarithm of the dimension of  $\mathcal{H}_{phys}^f$  is also of this order.

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<sup>4</sup> This claim can be verified directly for free fields, and for conformally invariant matter by conformally mapping  $F$  (up to a c-number anomaly term) to the Hamiltonian on the Einstein static universe. One expects it to hold in general.

To understand the full Hilbert space of physical states requires a complete theory of dS quantum gravity. While this is beyond the scope of our work, we note that the dimension of  $\mathcal{H}_{phys}^f$  is consistent with the conjecture [1, 2] that the full space of asymptotically (past and future) de Sitter states is of finite dimension, with entropy given by the Bekenstein-Hawking value  $S_{dS} = (R/\ell_p)^{D-2}$ . It is not a surprise that the number of weakly-coupled states is smaller than the de Sitter entropy. The argument is a standard one: At least when the number of species is small, the entropy of a given region of space is maximized when a black hole is present, which by definition is not a weakly-coupled phenomenon. To obtain the space of all asymptotically de Sitter states, we should include black holes (and possibly other non-perturbative gravitational states) up to the maximum size  $\sim R$ . The corresponding space of states  $\mathcal{H}^{dS}$  thus requires for its definition the full non-perturbative theory of gravity, and is expected to have dimension roughly  $e^{S_{dS}}$ .

To summarize, we have identified a space  $\mathcal{H}_{phys}^f$  for which perturbative gravity is weakly coupled. The construction of  $\mathcal{H}_{phys}^f$  proceeds by 1) beginning with the original Fock space  $\mathcal{H}_0$  describing the limit in which gravity is decoupled 2) restricting to the subspace  $\mathcal{H}^f$  satisfying  $F < f$  with respect to *some* foliation  $\mathcal{F}$  and 3) group averaging to produce the space of states  $\mathcal{H}_{phys}^f$ . The logarithm of the dimension of  $\mathcal{H}_{phys}^f$  is the size expected from local field theory, and is less than  $S_{dS}$ .

### C. Future (and past) asymptotically de Sitter states

While the space  $\mathcal{H}^{dS}$  may be the complete space of states describing a geometry that is asymptotically de Sitter to the past and future, in describing observables and measurement it appears useful to consider a much larger space. Specifically, if we are given a particular  $|\Psi\rangle \in \mathcal{H}^{dS}$ , physical predictions involve computing amplitudes of the form  $\langle\Psi'|\Psi\rangle$  for the state  $|\Psi\rangle$  to ‘look’ like the particular ‘descriptor state’<sup>5</sup>  $\langle\Psi'|$ . For example,  $\langle\Psi'|$  might describe the situation where you find yourself to be reading this particular sentence, as opposed to something else, at a particular instant in the evolution of the universe. Alternatively,  $\langle\Psi'|$  might describe fluctuations of a scalar field of some given magnitude, on an expanding spherical hypersurface with radius  $a$  satisfying  $a \gg R$ .

The reason this leads to a space larger than  $\mathcal{H}^{dS}$  is that a projection onto such a state does not in general commute with the requirement that the universe be both past and future asymptotically de Sitter. Let us consider the 2nd example above, and begin to evolve the descriptor state backwards in time from the given hypersurface. The fluctuations then blueshift by a large factor, and generically lead to a large gravitational back-reaction. Thus it is natural to think of  $\langle\Psi'|$  as lying outside  $\mathcal{H}^{dS}$ . We refer to the space of states described by data on a large expanding Cauchy surface as the ‘future asymptotically de Sitter’ space  $\mathcal{H}^{AsdS+}$ , and one may analogously define  $\mathcal{H}^{AsdS-}$ . Certain states in these enlarged spaces are also expected to play a role in describing physical cosmologies that are only asymptotically de Sitter in one direction.

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<sup>5</sup> An alternate perspective is to take the descriptor states to be  $|\psi'\rangle \in \mathcal{H}_0$ . Recalling that physical states  $|\Psi\rangle$  are best thought of as functionals on (a subspace of)  $\mathcal{H}_0$ , the desired amplitude  $\langle\Psi|\Psi'\rangle$  is just the action of  $|\Psi\rangle$  on  $|\psi'\rangle$ . This action admits a perturbative calculation if  $|\psi'\rangle$  is weakly coupled on some Cauchy surface.

Of course, many states in  $\mathcal{H}^{AsdS\pm}$  define the same functional on  $\mathcal{H}^{dS}$ . Since  $\mathcal{H}^{dS}$  is a Hilbert space, the independent functionals on  $\mathcal{H}^{dS}$  may be described as the subset  $\mathcal{H}^{dS} \subset \mathcal{H}^{AsdS\pm}$ . We will return to a discussion of  $\mathcal{H}^{AsdS\pm}$  in section IV, but for use in section III we note that one may wish to restrict  $\mathcal{H}^{AsdS\pm}$  to various other subspaces as well. For example, recall that the cutoff  $F < f$  leads to the finite-dimensional subspace  $\mathcal{H}_{phys}^f \subset \mathcal{H}^{AsdS+}$ . Likewise, to extend beyond this subspace, we could consider applying a flux bound on the flux  $F$  of (2.3) computed through an  $S^{D-1}$  at some radius  $a$  much larger than the neck radius, or with a different value of  $f$ . For given  $a$  and bound  $F < f$  this defines a new space,  $\mathcal{H}^{a,f} \subset \mathcal{H}^{AsdS+}$ , which is again of finite dimension. Note that there are inclusions  $\mathcal{H}^{a,f} \subset \mathcal{H}^{a',f'}$  for  $a < a'$ ,  $f < f'$ .

### III. OBSERVABLES, CUT-OFFS, AND LOCALITY

Though it is not our main focus here, it is appropriate to comment on how familiar (i.e., local) physics is to be recovered from the de Sitter-invariant physical states. Recall that this de Sitter invariance is a remnant of the diffeomorphism-invariance of the full gravitational theory. As a result, one expects the interesting dynamics to be *relational* [28, 29, 30, 31, 32, 33, 34, 35, 36, 37], and in particular to describe, e.g., the *relative* positions of various features of the quantum state. In the quantum dS context, this feature was pointed out in the original work [6], where the dS-invariance of states was first described.

Specifically, we expect to recover dynamics by using relational observables (see e.g. [34, 35, 36, 37, 38][19][39, 40, 41][23]; see also [42, 43, 44, 45, 46]). We refer the reader to [23] for a discussion of how, at least in appropriate states, ‘single integral observables’ of the form

$$\mathcal{O} = \int_M d^D x \sqrt{-g} A(x), \quad (3.1)$$

allow one to approximately recover local physics. An explicit example of this procedure was provided in [47] for a two-dimensional toy model, and general classes of one-dimensional models were analyzed in [19, 39]. Here  $M$  is the full spacetime manifold and  $A(x)$  is a local scalar. The resulting  $\mathcal{O}$  is diffeomorphism-invariant and thus, in our context, de Sitter invariant. When  $A(x) = C(x)D(x)$ , one may think of such observables as searching over all of  $M$  to find an appropriate detector  $D(x)$  at which the observable reports the value of some other field  $C(x)$ . If several detectors are found, the values of  $C(x)$  at each detector are summed.

One would like to call  $\mathcal{O}$  an “observable.” However, [23] pointed out a problem: perturbative quantum gravity about a dS background is *not* a context in which  $\mathcal{O}$  is a well-defined operator on either  $\mathcal{H}_0$  or  $\mathcal{H}_{phys}$ . Let us consider the case of  $\mathcal{H}_0$ , which is somewhat simpler. While it is straightforward to choose  $A(x)$  for which typical matrix elements of  $\mathcal{O}$  are finite, higher correlators involving  $\mathcal{O}$  diverge. Consider for example

$$\langle \psi_1 | \mathcal{O} \mathcal{O} | \psi_2 \rangle = \int d^D x \sqrt{-g} \int d^D y \sqrt{-g} \langle \psi_1 | A(x) A(y) | \psi_2 \rangle. \quad (3.2)$$

Over most of the integration region, the correlator  $\langle \psi_1 | A(x) A(y) | \psi_2 \rangle$  can be well approximated by its vacuum value. But the vacuum correlator is dS-invariant. Thus, the integrand does not change under the simultaneous action of a dS-translation on both  $x$  and  $y$ . The integral over this ‘center of mass’ direction diverges. In the ‘detector’ interpretation described



above, the divergence is due to the small probability that vacuum fluctuations give rise to virtual detectors in each small region of space, over a spacetime of infinite volume.

Recalling that quantum fluctuations in the dS-invariant vacuum are thermal fluctuations in any static patch, such fluctuations may be picturesquely thought of as due to the ‘Boltzmann brain’ phenomenon [48, 49, 50, 51], where a detector or observer can thermally fluctuate into existence and make an observation. Formally, such ‘virtual’ observers can confound an attempt to extract observations corresponding to detectors/observers not produced by this mechanism, and this may present an ultimate limit on constructing certain observables.

However, the cutoff prescriptions of the preceding subsection show how to regulate this divergence in a natural way. Since matrix elements of  $\mathcal{O}$  are finite, the correlators diverge only because an infinite number of intermediate states contribute. But our physical spaces  $\mathcal{H}_{phys}^{a,f}$  have only finitely many states. The problem is thus that  $\mathcal{O}$  does not preserve a given  $\mathcal{H}_{phys}^{a,f}$ . This happens because when  $A(x)$  acts far to the future of any dS-neck, it greatly changes the eigenvalue of the associated flux operator  $F$ . To remove the above divergence, let  $P_{phys}^{a,f}$  be the projector from  $\mathcal{H}_{phys}^{AsdS+}$  to  $\mathcal{H}_{phys}^{a,f}$ . Then  $\tilde{\mathcal{O}} = P_{phys}^{a,f} \mathcal{O} P_{phys}^{a,f}$  has well-defined correlators on  $\mathcal{H}_{phys}^{a,f}$ . The projection  $P_{phys}^{a,f}$  effectively limits the fluctuations to a finite integration volume and so, for sufficiently small  $a$  and  $f$ , ensures that rare vacuum fluctuations (aka Boltzmann brains) give a negligible contribution.

This ameliorates the paradoxical results of [51, 52], in the spirit of [53]. Having removed the problematic divergences, the stage is now set for the recovery of local physics (in an appropriate limit) along the lines described in e.g. [23]. More details of this limit will be investigated in future work.

One may ask how large the parameter  $a$  may be taken or, equivalently, up to what time past the neck such local observables may be recovered. To estimate this limit, note that the fluctuations into ‘virtual’ observers are suppressed by the observer’s mass  $m$ . The probability for such a fluctuation is proportional to  $\exp(-m/T_{dS})$ , where  $T_{dS} \sim 1/R$  is the de Sitter temperature. Any such observer should be contained in a causal patch, and thus the largest such observer should have mass bounded by  $f \sim m_p^{D-2} R^{D-3}$ . At time scales  $t \gtrsim R S_{dS}$ , the exponentially growing volume of the global picture produces a population explosion of such observers. We thus find that on these time scales, observations of even the largest (and thus heaviest) conceivable observers can become confounded by this miasma of Boltzmann brains. For small weakly coupled observers, the actual time scale is strictly less, though it can still be quite large when the cosmological constant is small.

#### IV. THE GLOBAL PICTURE OF ETERNAL DE SITTER SPACE

Thus far, we have studied in detail the states of perturbative gravity about a de Sitter background which are weakly coupled over the entire spacetime. Restriction to weak coupling imposed a cutoff on the energy flux through the de Sitter neck and resulted in a finite-dimensional space of physical states  $\mathcal{H}_{phys}^f$ , in which each state is precisely de Sitter invariant. The logarithm of the dimension of this space agrees to leading order with the entropy expected from local field theory in the absence of gravity.

We have also constructed observables on this space, from which local dynamics are to be recovered. The observables are relational, and their construction raised “Boltzmann brain” issues which required a (relational) infra-red cutoff. This cutoff is logically independent of

the limitation of physical states to  $\mathcal{H}_{phys}^f$ , and we argued that it could be associated with larger spaces of “descriptor states” such as  $\mathcal{H}_{phys}^{a,f}$ .

Which of these conclusions can be expected to hold in a more complete theory of eternal de Sitter quantum gravity? We now address each point in turn.

**De Sitter invariant states:** In the perturbative context, we emphasized that the de Sitter group acts as a *gauge* symmetry; it is part of the diffeomorphism gauge symmetry of low energy gravity and represents a redundancy in our description of the physics. In a full theory of eternal de Sitter, such spacetime concepts may or may not play a fundamental role. However, to the extent that any state may be said to be “asymptotically de Sitter to the past and future,” some notion of asymptotic de Sitter symmetry might be expected to be meaningful, and should again represent a redundancy of the description. We therefore might expect physical states  $\mathcal{H}_{phys}^{dS}$  of the full theory to be in an appropriate sense de Sitter invariant.

**De Sitter invariant observables:** By the same reasoning and to the same extent, observables of the full theory must also be de Sitter invariant. In this sense, meaningful dynamics will again be fully relational, and local observables will approximately arise from relational observables. For more discussion, see [23].

**The bound  $F < f$ :** For states satisfying this bound (and which are also locally weakly-coupled), perturbation theory is weakly coupled everywhere in the spacetime. It is therefore plausible that such states approximate states of the full theory, though we will return to this issue below. On the other hand, as has been pointed out by various authors (see e.g. [1]), significant violations of this bound should lead to gravitational collapse, where non-perturbative gravity is clearly relevant. It is an open question what the correct physics is in this regime. A proposed ‘nonlocality principle’ [54, 55] suggests that the non-perturbative gravitational dynamics of this regime has essentially nonlocal large-scale behavior. This is supported by studies of high-energy scattering [56] and by investigating locality through relational observables [23, 47], and could play an important role in explaining unitarity of black hole evaporation [54, 55, 57]. In the present context the boundary  $F \sim f$  in the field theory Fock space would represent a correspondence point where local field theory yields to such a new nonlocal theory needed for complete description of the quantum dynamics. In a flat background, this conjectured correspondence point is parametrized by the ‘locality bound’ proposed for two-particle states in [55, 58, 59] and in the N-particle case in [56]. If this perspective is correct, our bound  $F < f$  is thus naturally termed a ‘de Sitter locality bound.’ Holographic information bounds [60] have also been believed to point to where local quantum field theory breaks down. Assuming the expected connection between one bit of information and one quantum of energy apparently indicates that such bounds follow from locality bounds like our bound  $F < f$ , but the latter bounds appear more general in that they also apply to states with low entropy but high energy.

**Finite-dimensional state space:** We have constructed a space of states  $\mathcal{H}_{phys}^f$  which we expect are perturbatively close to actual interacting states corresponding to de Sitter space and, at least for a small number of species, we have argued that the number of such states is bounded by  $e^{S_{dS}}$ . These states are plausibly completed by nonperturbative states, involving black holes and other strong-gravitational effects, to give a space  $\mathcal{H}^{dS}$  of quantum de Sitter states, whose dimension is  $\sim e^{S_{dS}}$ . See [1, 2, 60] for related discussion.

**Large boosts and long times:** The group-averaging formalism requires integration over the full non-compact de Sitter group. This necessarily involves very large boosts, and the reader may ask if this computation can be controlled in perturbative gravity even for weakly

coupled states in  $\mathcal{H}_{phys}^f$ . Indeed, we find it plausible that subtle effects can be important in understanding the detailed action of large boosts. However, for given states satisfying  $F < f$ , the perturbative theory suggests that the contribution to the group averaging inner product falls off exponentially quickly in the boost parameter. In flat space, this is just the statement that two states of the same mass are orthogonal when their center-of-mass momenta differ by a large boost. This statement is expected to hold in the full theory, well beyond the perturbative regime. We will assume that it holds in the full theory of eternal dS as well.

It is important to ask in what regimes one might expect significant deviations from the results of local field theory. One place it is reasonable to expect deviations to arise is at the point where the observables of local field theory can no longer be recovered as limits of relational observables in the full theory. Our study of perturbative observables suggested one such limitation, where consideration of volumes larger than  $R^D e^{S_{dS}}$  led to typical observables corresponding to even the largest observers being confounded by vacuum fluctuations (‘Boltzmann brains’). This suggests that local field theory may break down for a complete description of global dS for times longer than  $t \sim RS_{dS}$ .

Other arguments have also identified this timescale as signaling a breakdown of local physics. Specifically, in parallel to the suggested breakdown [61][54, 55, 57] of local field theory in black hole contexts, [62] suggested that a global description breaks down at a time scale  $t \sim RS_{dS}$ . Ref. [57] outlined an argument for breakdown due to significant fluctuations on this time scale, and [62, 63] argued that this timescale also appears in an attempt to regulate inflation by exiting a slow rolling phase. This same time scale also arose in [64].

Together, these observations all point to the limit  $t < RS_{dS}$  on the time for which a local field theory description of the global geometry makes sense. At longer time-scales there may well be a valid description of a smaller region of de Sitter, *e.g.* a causal patch. Indeed, confounding by Boltzmann brains grows more probable with volume, so if one restricts to a causal patch, it appears that this effect becomes important only on a time scale  $t \sim Re^{S_{dS}}$ , which accords with observations of [65, 66] based on other considerations.

In either context, recall from section III that observables  $\tilde{\mathcal{O}} = P_{phys}^{a,f} \mathcal{O} P_{phys}^{a,f}$  are insensitive to such effects for moderate  $a$ , and plausibly for  $a \ll R \exp(S_{dS})$ . Similarly, group averaging calculations for states in  $\mathcal{H}_{phys}^f$ , and even in the larger spaces of descriptor states  $\mathcal{H}_{phys}^{a,f}$ , are insensitive to such effects for these values of  $a$ . We therefore expect computations of simple such correlators and matrix elements performed in the local field theory approximation (with gravitational perturbations) to reliably approximate results in the full theory. A similar conclusion should hold over similar spacetime volumes of more general shapes.

## V. DISCUSSION

A study of perturbative quantum gravity about a de Sitter background has motivated the global picture of eternal de Sitter space described above. Key features are a finite number of physical states, de Sitter invariance of *each* such state, relational dynamics, and an outline of the recovery of local dynamics as a limit for times  $t \ll RS_{dS}$ . Recovery over longer timescales may also be possible in limited regions of spacetimes, such as within a causal patch. The details of the local limit will be explored in future work.

One notes that our construction provides a counter-example to the claim of [66] that the de Sitter symmetries are not consistent with a finite entropy. Which of their assumptions

fail? It turns out that it is their description of the states and their dynamics. In [66] the contradiction arose from assuming that the dS generators act nontrivially on the finite number of physical states which were taken to be states of a single causal patch; the global picture was only used as a “thermofield double” formalism. But the contradiction disappears if the de Sitter generators annihilate the correct physical states, as we have argued.

One may note that a naïve perturbative approach based on local QFT still leads to a reduced density matrix  $\rho_S$  (describing e.g., the “south” causal patch) for which  $-Tr[\rho \ln \rho]$  is formally infinite. But such a computation has nothing to do with the dimension of our space of physical states. In particular, dynamics on our physical space is captured only by relational observables with appropriate cutoffs. Relational observables are also de Sitter invariant, and are not precisely restricted to the static patch surrounding a particular geodesic. Now, familiar local physics on a fixed background can often be recovered from relational observables by taking an appropriate limit. However, as argued in e.g. [23], a precise limit requires taking  $\ell_p$  to zero, so that  $S_{dS}$  diverges. One expects that any attempt to use relational observables to detect the entropy described in [66] while keeping  $\ell_p$  fixed would be subject to a cut-off which renders the entropy finite.<sup>6</sup> In the global picture, rather than finding a tension between finite entropy and dS-invariance, we find that dS-invariance (and the associated restriction to relational observables) cuts off the entropy at a finite value.

In short, it appears that some of the previously noted difficulties arising in a causal-patch picture of de Sitter space are artifacts of that picture. Another classic such issue is that of Poincaré recurrences. Since any time-translation invariant system with a finite number of states  $N \sim e^S$  experiences recurrences over long times of order  $e^S$ , it was argued in [3, 70] that similar issues should arise in the static patch of de Sitter. Now, as noted in [65], the operational status of such recurrences is unclear as, within this finite dimensional state space, it is not possible to construct detectors, memory devices, etc. that can meaningfully compare events separated on such long time scales. The detectors, memory devices, etc. will fall apart, stop working, or be destroyed much earlier. In addition, as we have described, on such timescales observables describing local measurements are confounded by fluctuations (section III). Nevertheless, there is at least a sense in which, despite the finite dimension of our state space, such recurrences do *not* arise at all in our setup. The point is that our dynamics is fully relational, and in particular that it is natural to specify dynamics relative to certain *global* features, such as presence of a ‘neck,’ i.e., a smallest  $S^{D-1}$ , or a larger  $S^{D-1}$  with given radius  $a$ . While global features experience quantum fluctuations, a thermal description is not appropriate; global features do not recur per se. Furthermore, dynamics specified relative to these global features effectively takes place on a time-dependent background; e.g., space is small at the neck but becomes arbitrarily large to both the future and past. In such a strongly time-dependent setting, with large asymptotic geometry, there is no reason for recurrences to arise.

Of course, by studying de Sitter space in a global description, we are working in a framework not apparently obeying the strongest forms of the complementarity conjecture. In those forms, this conjecture has been taken to imply that only a finite number of degrees of freedom describing a static patch make sense, and that physics outside this region is not described by independent degrees of freedom. Our picture is different.

Finally, we note that our considerations extend to portions of de Sitter space that are

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<sup>6</sup> In the black hole context, another physical motivation for a cut-off rendering this entropy finite was given in [67, 68, 69].

either produced through tunneling transitions or through prior FRW cosmologies, or which exit through termination of inflation. Though the discussion of constraints in such contexts is more subtle, dynamics will again be described by relational observables. Again, we expect to be able to effectively constrain to consideration of observables in a limited region, *e.g.* sufficiently near the region of transition into or out of de Sitter space in order to avoid being confounded by Boltzmann’s miasma. Either the flux projectors of section III, or more general relational constructions, could usefully play a role in this. We also expect considerations analogous to those above to lead to a finite number of states for many such spacetimes. Investigation of these issues is in progress.

## APPENDIX A: COUNTING PHYSICAL STATES

In the main text, we identified a space  $\mathcal{H}_{phys}^f$  for which perturbative gravity is weakly coupled. We also noted that these physical states can be obtained by acting with the group averaging map  $\eta$  on the states  $\mathcal{H}^{\mathcal{F},f}$  defined with respect to a fixed foliation. This fits with the intuition that every physical state should have a ‘gauge-fixed’ representative in the original Fock space. However, as a test of the formalism, we might ask to what extent states in a given  $\mathcal{H}^{\mathcal{F},f}$  are ‘gauge-equivalent’ to each other. That is, we may ask how much smaller is the physical space  $\mathcal{H}_{phys}^f$  as compared to the space of ‘seed’ states  $\mathcal{H}^{\mathcal{F},f}$ . Physically, global concerns arising from consistent coupling to gravity should not strongly affect the local physics of low-energy states, so the entropies associated with these two spaces should be comparable.

We now argue that these entropies are indeed comparable. We will construct a space of seed states  $\mathcal{H}_{seed}$  on which  $\eta$  has trivial kernel, but whose entropy agrees with that of  $\mathcal{H}^{\mathcal{F},f}$  to leading order in  $R/\ell_p$ . To proceed, recall that  $\mathcal{H}_{phys}^f$  can be obtained by group averaging any  $\mathcal{H}^{\mathcal{F},f}$  and let  $e^{S(f)}$  be the dimension of  $\mathcal{H}^{\mathcal{F},f}$ . Consider now the entropy  $S(f, J)$  of the ensemble of states in  $\mathcal{H}^{\mathcal{F},f}$  having total angular momentum  $J$  on the  $S^{D-1}$  spheres of our foliation. This entropy is maximized at  $J = 0$ . For large  $f$ , we are in the thermodynamic limit and the entropy of  $\mathcal{H}^{\mathcal{F},f}$  is dominated by states with  $J = 0$ . Thus,  $S(f, J = 0) \approx S(f)$  up to a logarithmic correction.

Furthermore, note that a generic  $J = 0$  state will also be in *local* thermodynamic equilibrium. In particular, consider any small region of the neck with size  $(R_c)$  small compared to  $R$ . To a good approximation, our state in this region will approximate a thermal state in flat spacetime. In this approximation we may say that the expected (spatial) momentum  $P_i$  vanishes in this region. Restricting our ensemble to states with  $P_i = 0$  in this region again makes only a logarithmic correction to the entropy. Doing so for a set of order  $(R/R_c)^{D-1}$  regions which cover the neck corrects the entropy by a factor which remains subleading for  $R_c \gg (R^2 \ell_p^{D-2})^{\frac{1}{D}}$ , where we have neglected a final logarithmic correction to the condition on  $R_c$ . We will refer to the resulting class of states as the set  $\mathcal{H}_{seed}$  of ‘locally zero-momentum seed states.’

We now argue that all locally zero-momentum seed states lead to independent physical states under group averaging. Consider two such seed states  $|\psi_1\rangle$  and  $|\psi_2\rangle$  which are orthogonal in  $\mathcal{H}_0$ . Since we required all seed states to have  $J = 0$ , spherical symmetry may be used to reduce the group averaging inner product (2.2) to the one-dimensional integral (see *e.g.* [12])

$$\int d\lambda \mu(\lambda) \langle \psi_1 | \exp(i\lambda B) | \psi_2 \rangle \quad (\text{A1})$$

where  $\mu(\lambda) > 0$  is an appropriate measure factor and  $B$  generates a ‘boost’ which fixes some  $S^{D-2}$  in the neck. Now consider a local region in which we required our seed states to satisfy  $P_i = 0$ , and which intersects this  $S^{D-2}$ . Since the energy density is non-zero, the boosted state  $U(B)|\psi_2\rangle$  represents an eigenstate of this local  $P_i$  with non-zero eigenvalue. It is therefore orthogonal to any  $P_i = 0$  eigenstate such as  $|\psi_1\rangle$ . In particular, we have  $\langle\Psi_1|\Psi_2\rangle = 0$ . On the other hand, since  $\mu(\lambda) > 0$  and we have argued that  $\langle\psi_1|\exp(i\lambda B)|\psi_2\rangle$  is essentially a  $\delta$ -function supported at  $\lambda = 0$ , this argument also demonstrates that the group averaging inner product (2.2) is positive definite on this class of states. It follows that  $|\Psi_1\rangle, |\Psi_2\rangle$  are linearly independent in  $\mathcal{H}_{phys}^f$ .

Let us summarize: For sufficiently large  $f$  we have found a class of seed states which are mapped into independent physical states by group averaging. We have also argued that the entropy of this class of seed states is  $S(f)$  up to logarithmic corrections. But  $S(f)$  is the entropy of the full space of states  $\mathcal{H}^{\mathcal{F},f}$  that we wish to group average. It follows that  $S(f)$  is also the entropy of  $\mathcal{H}_{phys}^f$ , so that our physical state space has the expected entropy.

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